SOLVING A REVERSE HEAT CONDUCTION PROBLEM BY ELECTRICAL SIMULATION

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A method of electrical simulation is proposed for determining the boundary conditions with respect to heat transfer when the temperatures in the body are known.

In recent years the methods of mathematical simulation, including electrical simulation, are being used more widely for the study of thermal processes. The solution of a heat conduction problem by electrical simulation, just as the solution of such a problem by analytical or numerical methods, can be reliable only if the data pertaining to given boundary conditions are correct.

As is well known, a steady-state temperature field can be described by the following second-order partial differential equation

$$\frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda(T)\frac{\partial T}{\partial y}\right]+\frac{\partial}{\partial z}\left[\lambda(T)\frac{\partial T}{\partial z}\right]=0, \tag{1}$$

which can generally be solved when the appropriate boundary conditions are given.

The boundary conditions have been classified in [1] into

$$\text{I kind} \qquad T_s = T_*, \tag{2}$$

$$\text{II kind} \qquad -\lambda(T)\left[\frac{\partial T}{\partial n}\right]_s = q_*, \tag{3}$$

$$\text{III kind} \qquad \alpha(T_s - T_a) = -\lambda(T)\left[\frac{\partial T}{\partial n}\right]_s, \tag{4}$$

$$\text{IV kind} \qquad T_{1s} = T_{2s},$$

$$-\lambda_1(T)\left[\frac{\partial T}{\partial n}\right]_{1s} = \lambda_2(T)\left[\frac{\partial T}{\partial n}\right]_{2s}. \tag{5}$$

The boundary conditions are established experimentally (by testing prototypes or physical models) and by calculation according to empirical formulas which generalize these test data. Often both methods do not appear feasible. For this reason, it is of particular interest to consider the solution of a reverse heat conduction problem, where the boundary conditions at the body surface are determined from temperature measurements (at a finite number of points inside the body). Usually one aims at determining the heat transfer coefficients at the body surface on the said body bulk temperatures and the ambient temperature. The problem can, however, be formulated broader, not just in terms of the third kind of boundary values, inasuch as often data pertaining to boundary conditions of the first, the second, and the fourth kind may prove to be just as important as the data pertaining to the heat transfer coefficients.

We will consider here a method of solving a reverse problem in steady-state heat conduction and the apparatus with which the surface temperature (boundary condition of the first kind), the thermal flux (boundary condition of the second kind), and the heat transfer coefficients during heat transmission across a contact (boundary condition of the fourth kind) can be found from the bulk temperatures of the body.

The method and the apparatus are based on the method of combinatorial circuits, which had been developed earlier for solving direct heat conduction problems [2] and which consists in using not only passive simulators (grids, electroconductive paper, etc.) but also electronic components with operational amplifiers.

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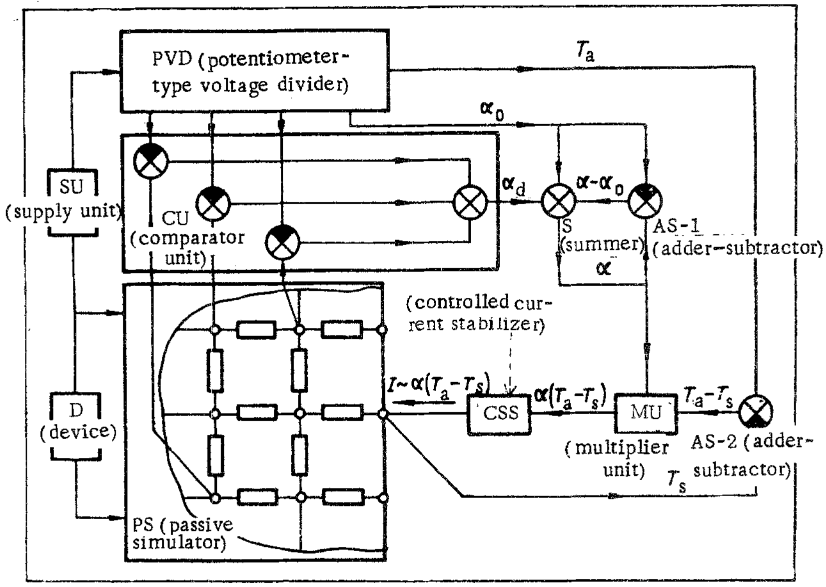

Fig. 1. Apparatus for determining the heat transfer coefficients.

Since all devices involved here have certain components in common, we will consider only one of it in greater detail, namely the one for determining the heat transfer coefficients at the body surface. As to the other devices, we will only indicate their distinguishing features so as not to overcrowd this article with repetitive descriptions of the purpose and the operating principle of components which also appear in that other device.

The device for determining the heat transfer coefficients (Fig. 1) consists of a passive simulator PS, a supply unit SU, a potentiometer-type voltage divider PVD, a comparator unit CU, two adder-subtractors AS-1 and AS-2, a summer S, a multiplier unit MU, and a controlled current stabilizer CSS coupled at the output to the passive simulator PS. The comparator unit CU includes adder-subtractors coupled at the output to the passive simulator PS and to the terminals of the voltage divider PVD, and a summer.

The device operates as follows. At the adder-subtractors of the comparator unit CU there appear signals from the node points of the passive simulator PS whose potentials have been fixed by the voltage divider and which correspond to the temperatures at those points. Difference pulses from the adder-subtractors are superposed in the summer of the comparator unit CU and the result α_d goes to one input of summer S, whose other inputs receive signals from the output of adder-subtractor AS-1 as well as a voltage proportional to some initial value α_0 from the voltage divider PVD. In AS-1 the voltage proportional to α_0 is subtracted from the output signal of summer S. This signal also goes to the multiplier unit MU, where it is multiplied by the output voltage from AS-2 proportional to the difference between the ambient temperature T_a and the temperature at the boundary point T_s. The AS-2 outputs are tied to the voltage divider PVD and to the boundary point of the simulator. The output signal from the multiplier unit MU goes to the input of the controlled current stabilizer CCS, where it is converted into a current proportional to the left-hand side of Eq. (4). The heat transfer coefficient is then measured, in the appropriate scale, with an instrument at the output of summer S.

For determining the thermal flux at the body surface, the schematic in Fig. 1 can be greatly simplified. One adder-subtractor and the multiplier unit can be eliminated along with the feedback coupling between the boundary node of the passive simulator PS and the controlled current stabilizer CCS. The thus modified device is shown schematically in Fig. 2. Here the total difference signal q_d goes from the comparator unit CU to the summer S where it is superposed on the voltage q_0 proportional to some initial value q_0 and on the signal coming from the adder-subtractor AS. Here the voltage proportional to q_0 is subtracted from the output signal of summer S. That latter signal is also the input signal to the controlled current stabilizer, which generates a current proportional to the thermal flux q at the boundary of the test body.

For determining the surface temperature of a body, the schematic in Fig. 2 can be further simplified, inasmuch as the controlled current stabilizer becomes superfluous here and the output of summer S

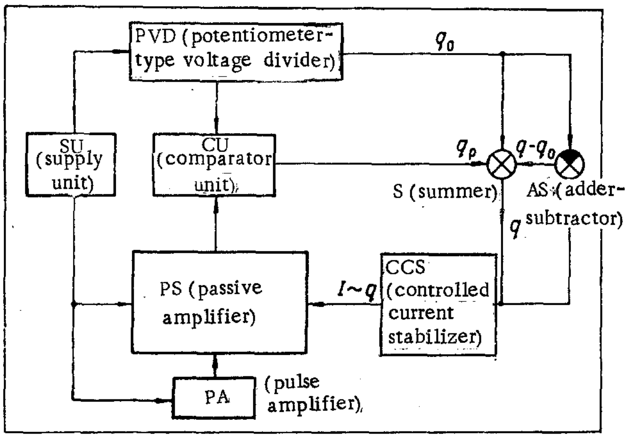

Fig. 2. Schematic diagram of apparatus for determining the thermal flux at the boundary.

can be connected directly to the boundary node of the passive simulator PS. The body surface temperature is proportional to the voltage at that boundary node.

The realization of boundary conditions of the fourth kind by the described method requires a rather complex arrangement, a not very practical one. In this case, evidently, servo systems of the type described in [3] are more effective.

With the aid of our proposed devices, we solved a few problems of determining the heat transfer coefficients at a body surface in the homogeneous case (infinitely large plate) and in the semiinfinite case (two-dimensional case). The latter problem is most interesting, inasmuch as a semiinfinite model is suitable for studying the heat transfer in inaccessible channels within massifs, in cooling systems for heat engines, etc.

The temperature field of a semiinfinitely large body shown in Fig. 3 has been obtained by simulating a direct heat conduction problem. The thermal conductivity of the material was $\lambda = 32.6$ W/m · °C. The ambient temperature and the heat transfer coefficient were respectively $T_h = 1073$°K, $\alpha_h = 11{,}450$ W/m^2 · °C on the hot side and $T_0 = 373$°K, $\alpha_0 = 5035$ W/m^2 · °C on the cold side. More explicitly are shown here the nodes whose temperatures served as the starting values for solving the reverse problem. In this way we determined the heat transfer coefficient in cooling ducts. This solution of the reverse problem yielded $\alpha_0 = 4710$ W/m^2 · °C, i.e., only 6.45% less than its value assumed for the solution of the direct problem. The dashed lines in Fig. 3 represent isotherms, and inside parentheses are indicated those node temperatures obtained by simulation after α_0 had been set automatically on the basis of the solution to the reverse problem. A comparison of both temperature fields indicates readily that they agree closely and, therefore, that the proposed devices yield a highly accurate solution to the reverse problem.

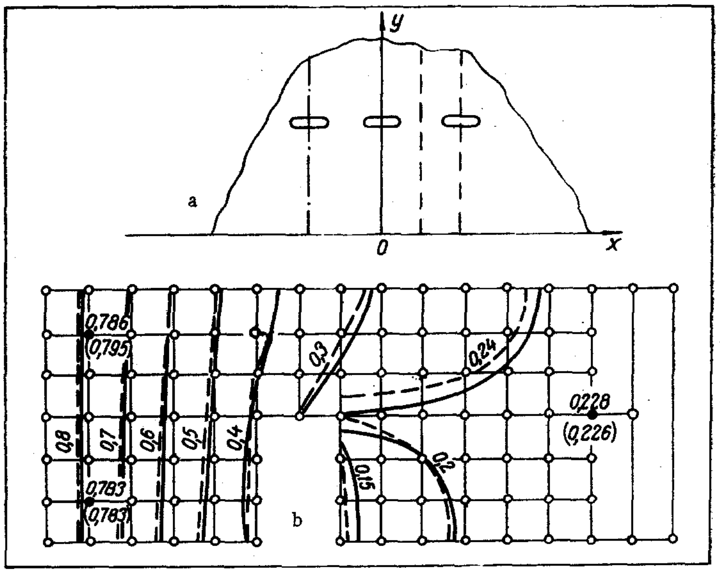

Fig. 3. Solution of the two-dimensional problem: (a) semiinfinitely large body (the simulated element is contained within dashed lines), (b) distribution of relative temperatures in the semiinfinitely large body ($\bar{\theta} = (T-T_0)/(T_h-T_0)$).

NOTATION

T	is the temperature;
λ	is the thermal conductivity;
α	is the heat transfer coefficient;
q	is the thermal flux;
x, y, z	is the Cartesian coordinates;
n	is the direction of external normal to the body surface.

Subscripts

s	refers to surface;
a	refers to ambient medium;
d	refers to difference (error) signal;
h	refers to hot side;
0	refers to cold side;
1, 2	refers to first and second body in contact.

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